FORM TP 2018298

TEST CODE **02134020**

MAY/JUNE 2018

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION C
Module 3
Answer BOTH questions.

5. (a) Use the substitution $u = x^4 + 2$ to determine $\int (x^4 + 2)^3 (4x^7)\, dx$. **[6 marks]**

(b) Calculate the area of the region lying between the parabolas $y = x^2$ and $x = \frac{1}{8} y^2$.

[6 marks]

(c) The equation of a curve is given as $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x$.

(i) Determine f'. **[1 mark]**

(ii) Determine f''. **[1 mark]**

(iii) Calculate the x coordinates of the stationary points of $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x$ and determine the nature of these stationary points.

[11 marks]

Total 25 marks

6. (a) A function f is defined as

$$f(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1} & x < 1 \\ 4x & x > 1 \\ 2 & x = 1. \end{cases}$$

(i) Determine whether or not the limit of f at $x = 1$ exists. **[4 marks]**

(ii) Determine whether f is continuous at $x = 1$. **[2 marks]**

(b) The parametric equation of a curve, g, is given by $x = 2 \cos \theta$ and $y = 3 - \sin \theta$.

(i) Determine $\frac{d}{,}$ g (x), in terms of θ. **[3 marks]**

(ii) Determine the equation of the normal to the curve at the point $\left(\sqrt{3}, \frac{5}{2}\right)$

[8 marks]

(c) The slope of a tangent of the curve, g, at any point (x, y) is equal to the product of the x-coordinate and the reciprocal of the y-coordinate of the point of tangency.

(i) Formulate an appropriate differential equation and find the equation of the curve family. **[5 marks]**

(ii) Hence, or otherwise, determine the equation of the curve given that it passes through the point (1, 3).

[3 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

SECTION B
Module 2
Answer BOTH questions.

3. (a) (i) Show that $\dfrac{\sin 2\theta - \cos 2\theta + 1}{\cos 2\theta + \sin 2\theta - 1} = \sec 2\theta + \tan 2\theta$. **[8 marks]**

(ii) Hence, or otherwise, determine the general solution of $\dfrac{\sin \theta - \cos \theta + 1}{\cos \theta + \sin \theta - 1} = 0$.

[5 marks]

(b) Given that $\cos A = \dfrac{3}{5}$ and $\sin B = \dfrac{3}{4}$, where A and B are acute angles, calculate

(i) $\sin 2A$ **[3 marks]**

(ii) $\cos (A + B)$. **[3 marks]**

(c) Solve the equation $\sin \theta - \sqrt{3}\, \cos \theta = 1$, for $-\pi \leq \theta \leq \pi$. **[6 marks]**

Total 25 marks

4. (a) The equations of the circle, C, and a straight line, L, are given by

$x^2 + y^2 - 2x + 2y + 1 = 0$ and $y = x - 3$, respectively.

(i) Determine the centre and radius of the circle, C. **[3 marks]**

(ii) Show that $(1, -2)$ is one of the points of intersection of the circle, C, and the straight line, L. **[6 marks]**

(iii) Determine the equation of the tangent to the circle, C, at the point $(1, -2)$.

[4 marks]

(b) Let $\mathbf{u} = \begin{pmatrix} s \\ 3 \\ s \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -2 \\ 6 \\ 4 \end{pmatrix}$ be two position vectors in R^3.

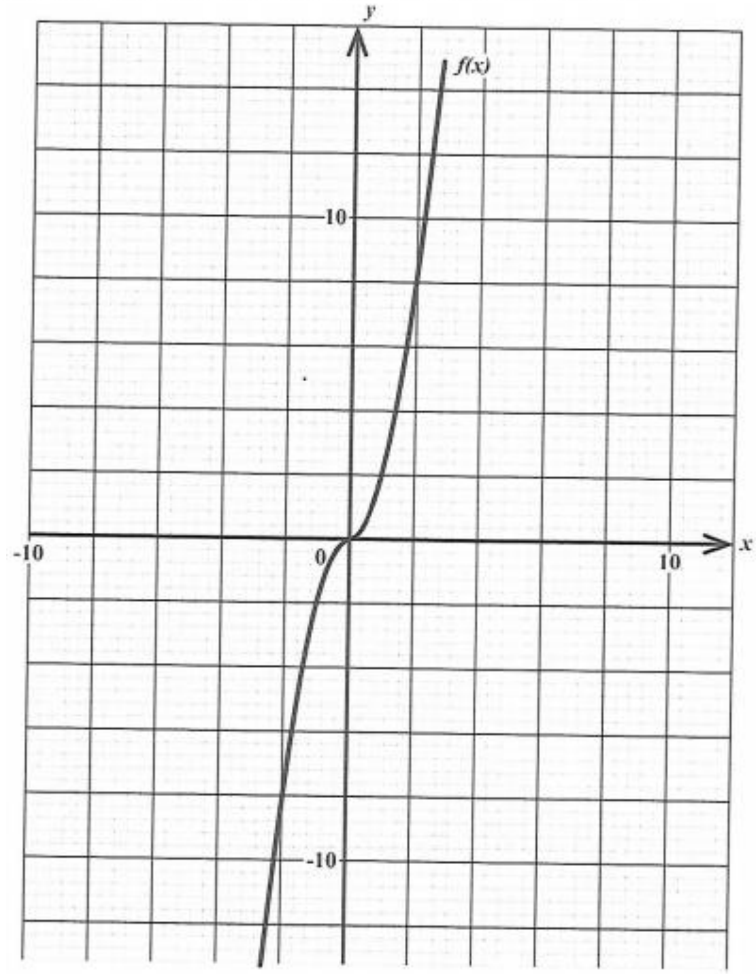
[9 marks]

Show that $\mathbf{u}$ and $\mathbf{v}$ are NOT parallel.

(c) Determine the vector equation of a plane which passes through the point (1, 3, 0) and which is perpendicular to the vector $2i + 4j + 5k$. **[3 marks]**

Total 25 marks

2. The diagram below shows the graph of the function $f(x)$.



(a) **On the diagram above,**

(i) sketch the inverse of $f(x)$. **[2 marks]**

(ii) Use a graphical method to show that f is bijective. **[3 marks]**

(b) (i) Prove that $|x-y| \leq |x-z| + |z-y|$ for all $x, y, z \in \mathbf{R}$. **[4 marks]**

(ii) Solve the inequality $|6x-2| + x^2 \leq 5$. **[8 marks]**

(c) Given that α, β and γ are the roots of the equation $2x^3 - x^2 + 1 = 0$, determine the equation whose roots are $\frac{1}{\alpha^2}$, $\frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$.

Note: $(\alpha\beta)^2 + (\alpha\gamma)^2 + (\beta\gamma)^2 = (\alpha\gamma + \beta\alpha + \gamma\alpha)^2 - 2\,\alpha\beta\gamma(\alpha + \beta + \gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

[8 marks]

Total 25 marks

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Let **p** and **q** be any two propositions. Complete the truth table below.

p	q	~ p	~ q	(p ∨ q)	~ (p ∨ q)	~ p ∧ ~ q
T	T					
T	F					
F	T					
F	F					

[4 marks]

(ii) Hence, state whether the statements ~ (**p** ∨ **q**) and ~ **p** ∧ ~ **q** are logically equivalent. **Justify your response.**

[1 mark]

(b) A binary operation ⊕ is defined on the set of real numbers, R, as $a \oplus b = 2a + 3b$ for all a, b in R.

(i) Calculate $5 \oplus 2$.

[1 mark]

(ii) Prove that ⊕ is closed in R.

[3 marks]

(iii) Determine whether ⊕ is commutative.

[3 marks]

(c) Given that $(2x + a)(x - 1)(bx + 1) \equiv cx^3 + 10x^2 - 2x - 10$ is a polynomial identity in x, calculate the values of a, b and c.

[7 marks]

(d) Solve the logarithmic equation $log_4 (2x + 2) - log_2 (x + 1) = 1$.

[6 marks]

Total 25 marks